# TIME SERIES APPLICATION OF A PERCENTILE ROLE DIFFERENTIATION INDICATOR

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# NAVAL POSTGRADUATE SCHOOL
## Monterey, California

# THESIS

TIME SERIES APPLICATION OF A
PERCENTILE ROLE DIFFERENTIATION INDICATOR

by

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March 1975

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# Time Series Application of a Percentile Role Differentiation Indicator

by

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ABSTRACT

This study is a first empirical application of Moore and Haga's probabilistic baseline generator of role differentiation in formal organizations. It employed time series data on a single sample to investigate the effects of variables other than organization growth on the evolution of organization complexity under an assumption that both members and roles were interchangeable. The resulting statistic was a percentile role differentiation indicator (PRDI-V) that was reliable for organization sizes above approximately 75. The theoretical interest and practical organizational auditing utility of the PRDI are discussed.

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# I. INTRODUCTION

As large bureaucracies become more pervasive in everyday life, sociologists have increased their interest in the causal relationships between organization size and complexity. Blau (1970) hypothesized that increased size promoted increased complexity at a monotonically decreasing rate. Mayhew et al. (1972) extended Blau's notion by suggesting an algorithm that generated the number of logically possible arrangements of roles within an organization of given size. Moore and Haga (1975) developed a set of size-complexity algorithms for five general assumptions about the nature of people and the nature of the roles in organizations.

This study examines the utility of one of the percentile role differentiation indicators (PRDI) developed by Moore and Haga with longitudinal data from a single sample.

## II. LITERATURE REVIEW

Blau (1970) specified that formal organizations subdivide responsibilities in order to do their work. His research on structural differentiation was based upon data from 53 employment security agencies in the United States. His basic hypothesis was that, "Increasing size generates structural differentiation in organizations along various dimensions at decelerating rates." His alternative formulation of this proposition was broken into three parts: (1) large size promotes structural differentiation, (2) large size promotes differentiation along several different lines, and (3) the rate of differentiation declines with expanding size. Blau's second generalization was that structural differentiation in organizations enlarges the administrative component. However he determined that, "the economies of scale exceed the costs of differentiation, so that large organizations, despite their greater structural complexity, require proportionately less administrative manpower than small ones."

Meyer (1972) noted that some investigators, (Zelditch and Hopkins, 1961), believed organizations could be studied without direct reference to size while others, (Blau, 1970; Blau and Schoenherr, 1971), based their theories of complexity almost entirely upon the effects of size. Meyer pointed out that although there may be correlations among variables, such as size, number of administrative roles, etc., causal

interpretations are hazardous because the time ordering of variables is not known. He examined causal relationships among four parameters; (1) size, (2) number of major subunits, (3) number of levels of hierarchy, and (4) number of supervisors in an organization. He stated his most important finding as "one cannot underestimate the impact of size on other characteristics of organizations. Size must always be controlled when analyzing relationships among properties of organizations. But constraints and boundary conditions inherent in cross-sectional data often inflate or distort the effects of size; hence, size cannot truly be taken into account in studies that examine sets of organizations at only one point in time."

Hendershot and James (1972) supported Blau's propositions concluding that there was a more general negative relationship between organization size and the administrative-production ratio. Their study which was based upon supervisors and teachers in American school districts also suggested that the effect of organization growth on the administrative-production ratio depended on the magnitude of growth. Slow growth, it appeared, tended to produce an increase in the ratio, but rapid growth tended to produce a decrease.

## III. METHODOLOGY

### A. SAMPLE

The data used in this study was gathered from a central California unified school district. From establishment in 1896 until 1945, it operated as two school systems, one for a high school and the other for elementary grades. A single elected school board presided over both systems. Separate financial records and minutes of board meetings were kept for the two districts. In 1946 the districts and the record keeping were consolidated and the board membership was expanded from three to five.

### B. DESIGN OF DATA COLLECTION

Minutes of the district's board for the years 1904 through 1962 provided data on the structure of the school system. Financial records were also used. Telephone directories were used in later years to develop the necessary organization charts. Available copies of the "Principal's October Report" document were also used. Reports to the state concerning average daily attendance, total receipts and expenditures, capital outlay, total teachers' salaries, and total administrator salaries were reviewed. In addition some of these reports indicated the number of administrators by category, supervising and non-supervising and the total number of elementary and high school teachers. The two high school

principals for the period 1952-1972 and 1921-1948 were contacted and additional information provided. Three fiscal years 1956 through 1958, were excluded from the sample because of the lack of reliable information concerning the number of high school roles in existence and teachers employed during this period. Part-time and full-time positions were differentiated. Lists of all positions for each year were prepared with the number of people in an occupation on either a full-time or a part-time basis. An individual could hold a full-time and part-time position simultaneously, e.g., full-time teacher and part-time librarian.

## C. ANALYSIS STRATEGY

Mayhew _et_ _al_. (1972) did not make explicit what assumptions were made concerning the nature of the people and the roles they fulfilled. There was an implication that they assumed the general pyramid structure commonly outlined in reference to bureaucratic organizations.

Moore and Haga developed five explicit assumptions concerning the interchangeability of the people fulfilling those roles. They define a role as "...a set of people doing the same work such that the persons within a role have no hierarchical distinctions among themselves ... that members of a role are freely interchangeable within that role without effecting the structure of the organization." They make five interchangeability assumptions concerning roles and people,

developing an algorithm for each. These five assumptions are not the only ones that can be developed but are the most general.

If two people who are in different roles switch places and a new legitimate arrangement of the organization occured, then the people would be said to be "non-interchangeable." If the people are "non-interchangeable," when a switch of people is made it would create another logical arrangement of placing $\underline{s}$ people in $\underline{k}$ roles. Consequently, this arrangement would be counted and added to $\underline{n}$ the number of logically possible arrangements. Conversely if two people, in different roles, can be switched without increasing $\underline{n}$ then the people are said to be "interchangeable." The roles of an organization are defined as being "non-interchangeable" if they have order among themselves whether actual or arbitrary. If the order of roles A-B-C was changed to A-C-B and an increase in the $\underline{n}$ logical ways of arranging the organization of size $\underline{s}$ occured, then the roles would be said to be "non-interchangeable." If this change in order of roles from A-B-C to A-C-B did not increase $\underline{n}$, then the roles would be said to be "interchangebable." (See Figure 1.)

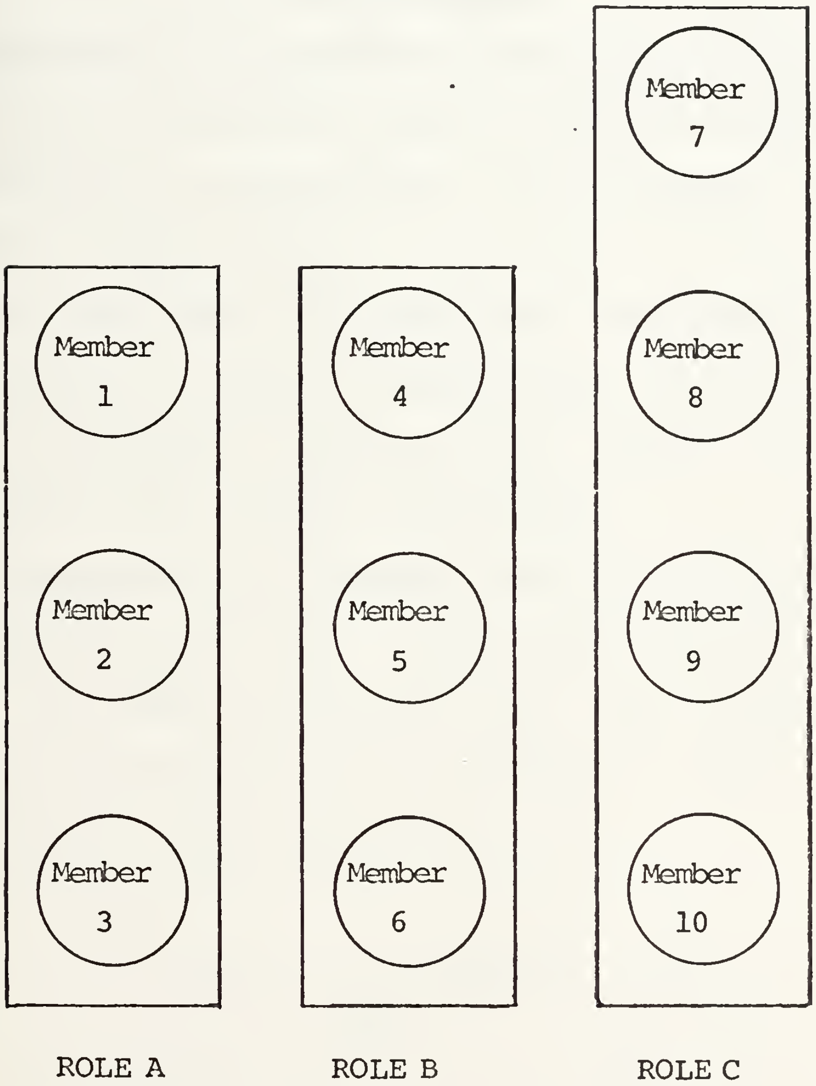
The five general assumptions are:

Assumption I: roles are not interchangeable/people are not interchangeable

Assumption II: same size roles are interchangeable/people are not interchangeable

FIGURE 1

Hypothetical organization with roles A-B-C and people one through ten




Organization A B C

Assumption III: roles are not interchangeable/people are interchangeable

Assumption IV: same size roles are interchangeable/people are interchangeable

Assumption V: roles are interchangeable/people are interchangeable.

Algorithms for generating the $n$ logical possible arrangements were developed for each of the five assumptions. The one developed for Assumption V, the assumption used in this analysis is given below.

Moore and Haga (1975) developed this probabilistic baseline generator to study organizational role differentiation against an absolute standard of all logical possible arrangements for a given organization.

By making both individuals and roles interchangeable the sequence $(r_1 \ldots\ldots r_k)$ will be restricted to a nonincreasing order. The number of nonincreasing order arrangements of length $k$ that sum to $s$ are represented by $N_s^{(k)}$. The generating function for $N_s^{(k)}$ is

$$p(t,k) = \sum_{s=1}^{\infty} N_s^{(k)} t^s = \frac{t^k}{\prod_{i=1}^{k} (1-t')} .$$

Analysis of p(t,k) yields the following:

(1) $N_s^{(k)} = 0 \quad \text{if } s < k$

(2) $N_s^{(1)} = 1$ if $s = 1,2,\ldots$

(3) $N_s^{(k)} = N_{s-1}^{(k-1)} + N_{s-k}^{(k)}$ if $k \leq \frac{s}{2}$

(4) $\sum_{j=1}^{k} N_s^{(j)} = N_{s+k}^{(k)}$

(5) $N_s^{(k)} = N_{2s-2k}^{(s-k)}$ if $k > \frac{s}{2}$

Results (3) and (5) above are recurrence relations that are useful in generating a probability table for $N_s^{(k)}$.

By noting that $N_{2s}^{(s)} = \sum_{j=1}^{s} N_s^{(j)} = p_s$ the generating function for $p_s$ is obtained as

$$p(t) = \sum_{s=1}^{\infty} P_s t^s + 1 = \sum_{k=1}^{\infty} p(t,k) = \frac{1}{\prod_{i=0}^{\infty} (1-t^i)}$$

This generating function has been used by Gupta (1935,1937) to generate a table of $p_s$ and to demonstrate that

$$N_{2s}^{(s)} = \frac{1}{\sqrt{48s^2}} e^{\pi \frac{2s}{3}}$$

for organizations of large sizes. In general

$$N_s^{(k)} = [\frac{k(s-k)}{(k!)^2} u_{k-2}^{(s)}] + 1$$

where the braces denote the integer part of the value enclosed and $u_L^{(s)}$ is an Lth order polynomial in s. For example, $u_0(s) = 1$, $u_1(s) = s+3$, and $u_2(s) = s^2+7s+28$.

Assumption V fits the school system in this study. Consider the staffing of a school system from scratch. There would be k roles to be filled with s people. The s people and their skills would be matched as best as possible with the experience requirements of the k roles. Once the roles were filled and the organization running the people would not be allowed to switch roles. A janitor would not ordinarily be allowed to switch with a teacher. Therefore under Assumption V, such a switch would not be allowable, and does not increase the n logically possible arrangements.

This study is one of the first empirical applications of the utility of Moore and Haga's PRDI.

$$p(k,s) = P[K=k|s]$$

is the probability of assigning s people to k roles under Assumption V. Assuming that every assignment is equally likely, then p(k,s) is the number of ways of assigning s people to k roles divided by the total number of assignments

under the assumption. The method used to evaluate the structural differentiation of the organization with k roles was by noting the probability that a randomly chosen organization of size s could have had fewer than k roles. Thus

$$M_1(k,s) = \sum_{i=1}^{k-1} p(i,s) = P[K < k|s]$$

where $M_1(k,s)$ is the percentile indicator of differentiation ranging from 0% to 100%.

In order to calculate the PRDI-V for a particular k for a particular size the developed computer program was run for each of the 67 years of available data. Next, the number of full-time roles minus one was used to enter the generated table.[1] The number obtained from the table was then divided by the last number in the table. The resulting statistic is the PRDI-V. For example in year 1932 there were 43 employees working in 19 roles in the sample organization: s = 43 and k = 19. The table generated for s = 43, with k = 19 developed the PRDI-V 0.884. Therefore 88.4% of the logical ways, assuming Assumption V above, of arranging 43 employees would have generated a number less than the 19 actual full-time roles which existed.

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[1]If the number representing the number of full-time roles, instead of a number one smaller, had been used to enter the table then rather than a percentile or "less than" ( $<$ ) number being calculated a "less than or equal to" ( $\leq$ ) number would have been calculated.

## IV. RESULTS

The PRDI-V values developed for each year are shown in Table I. Because of the high volatility of the numbers for the early years, when the organization was small, only the 23 available years between 1948 and 1973, inclusive, were used. This volatility is because the PRDI is unstable for small sample sizes of discrete data. A correlation matrix, Table II, was developed between ten different variables (see Appendix A) for the 23 years. Other than for variable six for which the results were not significant at the .005 level, the administrative/production (A/P) ratio (variable 2) correlated with higher numbers with the eight other variables, variables three through ten, than did the PRDI-V (variable 1). PRDI-V (variable 1) and size (variable 4) were correlated with 0.81 which implied that the PRDI-V might not be size invariant. In order to determine if PRDI-V was size invariant, a linear regression was run on the ten variables with PRDI-V as the dependent variable. Two of the nine other variables entered the regression. In the first step of the regression year (variable 3) entered. In the second step, percentage of direct output line supervisor (variable 6) entered the regression. None of the other variables entered the regression at the .01 level of significance.

TABLE I

Listing of PRDI-V by Year

| YEAR | PRDI-V | YEAR | PRDI-V | YEAR | PRDI-V |
|---|---|---|---|---|---|
| 1904 | .71 | 1928 | .65 | 1952 | .75 |
| 1905 | .61 | 1929 | .79 | 1953 | .87 |
| 1906 | .61 | 1930 | .90 | 1954 | .77 |
| 1907 | .56 | 1931 | .88 | 1955 | .76 |
| 1908 | .52 | 1932 | .88 | 1959 | .82 |
| 1909 | .52 | 1933 | .88 | 1960 | .84 |
| 1910 | .56 | 1934 | .92 | 1961 | .79 |
| 1911 | .81 | 1935 | .93 | 1962 | .71 |
| 1912 | .62 | 1936 | .88 | 1963 | .83 |
| 1913 | .72 | 1937 | .83 | 1964 | .86 |
| 1914 | .75 | 1938 | .78 | 1965 | .81 |
| 1915 | .80 | 1939 | .75 | 1966 | .86 |
| 1916 | .83 | 1940 | .84 | 1967 | .94 |
| 1917 | .86 | 1941 | .73 | 1968 | .93 |
| 1918 | .80 | 1942 | .71 | 1969 | .88 |
| 1919 | .80 | 1943 | .82 | 1970 | .85 |
| 1920 | .78 | 1944 | .71 | 1971 | .89 |
| 1921 | .81 | 1945 | .76 | 1972 | .89 |
| 1922 | .73 | 1946 | .69 | 1973 | .95 |
| 1923 | .85 | 1947 | .85 | | |
| 1924 | .90 | 1948 | .66 | | |
| 1925 | .88 | 1949 | .70 | | |
| 1926 | .88 | 1950 | .60 | | |
| 1927 | .79 | 1951 | .71 | | |

TABLE II

Correlation Matrix, PRDI-V Without Lag

| VARIABLES | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.00 | .82* | .83* | .81* | .81* | -.03 | .72* | .75* | .78* | .78* |
| 2 | | 1.00 | .97* | .95* | .83* | -.39 | .90* | .89* | .95* | .87* |
| 3 | | | 1.00 | .99* | .82* | -.41 | .87* | .88* | .97* | .94* |
| 4 | | | | 1.00 | .76* | -.41 | .84* | .87* | .94* | .95* |
| 5 | | | | | 1.00 | -.31 | .83* | .65* | .84* | .66* |
| 6 | | | | | | 1.00 | -.38 | -.30 | -.47 | -.34 |
| 7 | | | | | | | 1.00 | .64* | .93* | .69* |
| 8 | | | | | | | | 1.00 | .76* | .91* |
| 9 | | | | | | | | | 1.00 | .82* |
| 10 | | | | | | | | | | 1.00 |

NOTE: * $p < .005$

See Appendix A for corresponding variable names.

This generated a linear regression of

$$(\text{PRDI-V}) = (.011)(\text{year}) + (.030)(\%\ \text{direct line supv.}) - (.023)$$

with multiple $R^2 = .800$. In other words, 80% of the variance of PRDI-V was explained by the variance in variables three and six. Consequently, the apparent correlation between PRDI-V (variable 1) and size (variable 4) was spurious meaning that PRDI-V was size invariant.

PRDI-V was investigated for a lagged effect by the independent variables across time. Correlation matrix (Table III) was generated between variables two through ten of a particular year and the PRDI-V of the succeeding year. With the exception of % non-direct staff (variable 8) the significant correlations with PRDI-V without lag were greater than those with one year lagged PRDI-V.

TABLE III

Correlation Matrix, One Year Lagged PRDI-V

| VARIABLES | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.00 | .78* | .79* | .75* | .70* | -.29 | .63* | .76* | .75* | .74* |
| 2 | | 1.00 | .98* | .96* | .81* | -.33 | .88* | .89* | .94* | .92* |
| 3 | | | 1.00 | .99* | .80* | -.36 | .86* | .86* | .97* | .95* |
| 4 | | | | 1.00 | .74* | -.37 | .84* | .86* | .95* | .96* |
| 5 | | | | | 1.00 | -.25 | .80* | .61* | .82* | .66* |
| 6 | | | | | | 1.00 | -.32 | -.24 | -.43 | -.32 |
| 7 | | | | | | | 1.00 | .59* | .91* | .71* |
| 8 | | | | | | | | 1.00 | .74* | .92* |
| 9 | | | | | | | | | 1.00 | .86* |
| 10 | | | | | | | | | | 1.00 |

NOTE: * $p < .005$

See Appendix A for corresponding variable names.

# V. CONCLUSIONS

This study, as an initial examination, demonstrated the utility of the Moore and Haga PRDI. Previous efforts by Mayhew et al. had developed the concept of viewing organization complexity from the point of a logical baseline. However, with this previous baseline it was impossible to make (1) specific assumptions concerning the nature of people and the roles in which they worked and (2) calculations for organizations with a size greater than 70. These weaknesses have been eliminated with the PRDI.

If no other variable other than size were affecting the values, from a theoretical viewpoint, the PRDI would necessarily be relatively constant within a given sample if it is to have any meaning. If this were not the case it would be impossible to use the PRDI to study non-size influences across time.

The correct PRDI, among the five developed by Moore and Haga, should develop values within a sample with a mean of approximately 0.5 if the samples were generated totally at random. Data indicated this was not the case in this sample. Further work should be done by testing each of the PRDIs against a specific sample. However, the one which would be the most useful would not necessarily be the one for which the values had a mean approximating 0.5 but rather the one for which the values deviated the least around a particular

mean, irrespective of its value. To accomplish this various organizations should be stratified into categories based upon similar factors, e.g., financial, geographical, markets, products, etc., and then all five of the PRDIs run against these stratifications. Then a determination could be made as to which PRDI had the least variance for which cluster (by a specific factor) of organizations.

The procedures above are "categorizing," however, given an accurate selection of the "right" PRDI, it could be used as a predictor within applicable limits. It would also have potential as an "auditing" tool in that it could point to organizations which generated particular PRDI values different from those which would be theoretically assumed so that then an analysis could be made of the organization to determine the specific, unique factor(s) that might be causing the difference.

This study was a first tentative validation of the PRDI concept on an empirical basis. It was with a large time-series sample and found PRDI to be generally reliable (not found to be very reliable when the organization was of small size). The time-series was of sufficient length that it encompassed several external, "macro" events, e.g. changes in financing, recessions, wars, etc.

This limited validation suggests the usefulness of other studies of the PRDIs other than PRDI-V and in conjunction with A/P ratios and studies with other types of organizations.

## APPENDIX A

### List of Variables

| Variable Number | Description |
|---|---|
| 1 | PRDI |
| 2 | A/P ratio, based upon number of employees |
| 3 | year |
| 4 | size, number of full-time employees |
| 5 | number of full-time direct system output roles |
| 6 | % direct output line supervisors |
| 7 | % clerical workers |
| 8 | % non-direct staff |
| 9 | total annual system expenses, in dollars |
| 10 | number of students |

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